The Thirteenth International Symposium on Artificial Life and Robotics 2008(AROB 13th '08),
B-Con Plaza, Beppu, Oita, Japan, January 31-February 2, 2008

Fuzzy Sliding Mode Control for a Robot Manipulator

Ha Quang Thinh Ngo*, Jin-Ho Shin[O], and Won-Ho Kim[O]

*Department of Intelligent System Engineering, Graduate School of Dong-eui University, Busan, Korea
[O]Department of Mechatronics Engineering, Dong-eui University, Busan, Korea
(Tel: +82-51-890-2257, Fax: +82-51-890-2255)
(thinhpfiev2003@yahoo.com, jhshin7@deu.ac.kr, kwh@deu.ac.kr)

Abstract: This work presents the design of a robust control system using a sliding mode controller that incorporates a fuzzy control scheme. The presented control law superposes a sliding mode controller and a fuzzy logic controller. A fuzzy tuning scheme is employed to improve the performance of the control system. The proposed fuzzy sliding mode control (FSMC) scheme utilizes the complementary cooperation of the traditional sliding mode control (SMC) and the fuzzy logic control (FLC). In other words, the proposed control scheme has the advantages which it can guarantee the stability in the sense of Lyapunov function theory and can ameliorate the tracking errors, compared with the FLC and SMC, respectively. Simulation results for the trajectory tracking control of a two-link robot manipulator are presented to show the feasibility and robustness of the proposed control scheme.

Keywords: fuzzy sliding mode control, fuzzy logic control, stability, robot manipulator, tracking control.

I. INTRODUCTION

In real dynamical systems, it is impossible to avoid uncertainties due to imperfect modeling and due to the environment factors such as temperature, pressure and other external disturbances. There are many solutions, however, a more general solution is SMC since it has, such as good control performance for nonlinear systems, applicability to multi-input-multi-output systems. The most significant property of an SMC is its robustness. SMC nowadays enjoys a wide variety of application areas, such as in robotics [1], in process control [2], in dc motor control [3] and so on.

Fuzzy logic in control design has been commonly used for numerous electrical system, robotic system and mechanical system in recent years. Fuzzy controller is often designed from an intuitive standpoint rather than from a stability standpoint [4]. This has several advantages that fuzzy controller is more understandable and more easily modified than other controller.

In fact, a pure SMC suffers from the following disadvantages [5]. First, there is the problem of chattering, which is the high-frequency oscillations of the controller output, brought about by the high speed switching necessary for the establishment of a sliding mode. Second, an SMC is extremely vulnerable to measure noise since the input depends on the sign of a measured variable that is very close to zero. Third, the SMC may employ unnecessarily large control signals to overcome the parametric uncertainties. To attenuate these difficulties, several methods are proposed [6], the most popular one is boundary layer approach. Nonetheless, boundary layer controller does not guarantee asymptotic stability but rather uniform ultimate boundedness [7]. Some boundary layer width modification techniques to improve tracking precision are discussed in [8].

The objective of this paper is to propose a controller that can alleviate the chattering and reduce the tracking errors. The controller should overcome the uncertainties such as parameter variations, unknown friction forces and external disturbances. Section 2 provides the dynamic model of a robot. The main part for a design of the proposed controller is presented in section 3. Finally, simulation results and conclusions are given in section 4 and 5, respectively.

II. Dynamic Model of a Robot Manipulator

The dynamic equation of motion for a robot manipulator is described in joint space.

$$M(q)\ddot{q} + B(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) = w(t) + u \qquad (1)$$

where $M(q)$ is a symmetric positive definite inertia matrix, $B(q,\dot{q})$ is a matrix of coriolis and centrifugal torques, $G(q)$ is a vector of gravitational torques, $F(\dot{q})$ represents a vector of friction torques, $w(t)$ is a bounded external disturbance vector, u is a control input torque vector, and q is the joint angular position vector of the robot.

Let the following superscript "o" represent the nominal parameter values of the robot and the symbol "Δ" denote the uncertain parameter values.

$$M = M^0 + \Delta M, \qquad B = B^0 + \Delta B, \qquad G = G^0 + \Delta G, \qquad F = F^0 + \Delta F, \tag{2}$$

The robot dynamics and the uncertainty terms have the following properties and assumption.

Property 1: Using a proper definition of the matrix B, $\dot{M}(q) - 2B(q,\dot{q})$ is skew-symmetric and satisfies

$$x^T[\dot{M}(q) - 2B(q,\dot{q})]x = 0, \ \forall x \in R^n \tag{3}$$

Assumption 1: It is assumed that there exist known positive constants M_{ij}^m, B_{ij}^m, $f_i^{m_1}$, $f_i^{m_2}$, G_i^m and w_i^m such that the each elements of the matrices and vectors $\Delta M_{ij}, \Delta B_{ij}, \Delta F_i$, ΔG_i for the uncertain robot parameters and the disturbances w_i are bounded as

$$|\Delta M_{ij}| \le M_{ij}^m, \qquad |\Delta B_{ij}| \le B_{ij}^m |\dot{q}_i|, \qquad |\Delta G_i| \le G_i^m, \quad |\Delta F_i| \le f_i^{m_1} + f_i^{m_2}|\dot{q}_i|, \quad |w_i| \le w_i^m \tag{4}$$

where $|\cdot|$ represents the absolute value.

III. FUZZY SLIDING MODE CONTROL

Let q_d represent the desired position and the position error is $e = q - q_d$.

The switching surface s is chosen as $s = \dot{e} + Ce$, where C is a positive constant diagonal gain matrix. The gain C determines the rate of response of the system, and the aim of the control is to force the motion of the system to be moved along the intersection of the switching plane in $s = 0$.

Differentiating s with respect to time,

$$\dot{s} = C(\dot{q} - \dot{q}_d) + \ddot{q} - \ddot{q}_d \tag{5}$$

Multiplying the matrix M to (6) and substituting it into (1), we have

$$M\dot{s} = MC(\dot{q} - \dot{q}_d) + M\ddot{q} - M\ddot{q}_d = w + u - B\dot{q} - G - F + MC\dot{e} - M\ddot{q}_d \tag{6}$$

Consider a Lyapunov function candidate

$$V = \frac{1}{2} s^T M(q,\dot{q}) s \tag{7}$$

Differentiating V with respect to time,

$$\dot{V} = \frac{1}{2} s^T \dot{M}(q,\dot{q}) s + s^T M(q,\dot{q})\dot{s} = s^T B s + s^T(M\dot{s}) \tag{8}$$

Inserting (6) into the above equation

$$\dot{V} = s^T B s + s^T[u + w - B\dot{q} - G - F - M\ddot{q}_d + MC\dot{e}] \tag{9}$$

The proposed controller here is designed by combining sliding mode control with fuzzy logic control.

The presented robust fuzzy sliding mode controller is designed as the form shown in [9]

$$u = u_0 + u_s + U_F \tag{10}$$

$$u_0 = M^0\ddot{q}_d + B^0\dot{q} + G^0 + F^0 - M^0 C\dot{e} - B^0 s, \tag{11}$$

$$u_S = -K \otimes sat\left(\frac{s}{\Phi}\right), \tag{12}$$

$$K = M^m |C\dot{e} - \ddot{q}_d| + B^m |\dot{q}||s - \dot{q}| + G^m + f^{m_1} + f^{m_2}|\dot{q}| + w^m + \eta \tag{13}$$

where u_0 is an equivalent control input, u_s is a robust input overcoming the uncertainties and uses the saturation function to alleviate the chattering, and U_F is a fuzzy control input. $sat(s/\Phi) = Vector[sat(s_i/\Phi_i)]$,

$sat\left(\frac{s_i}{\Phi_i}\right) = \begin{cases} \mathrm{sgn}(s_i) & if \ |\frac{s_i}{\Phi_i}| > 1 \\ \frac{s_i}{\Phi_i} & if \ |\frac{s_i}{\Phi_i}| \le 1 \end{cases}$, The symbol "$\otimes$", which is called a vector multiplication, is defined as $u_{s_i} = -K_i sat(\frac{s_i}{\Phi_i})$. $K \in \Re^n$ is a robust gain overcoming the uncertainties. The bound values $M^m = Matrix[M_{ij}^m]$, $B^m = Matrix[B_{ij}^m]$, $G^m = Vector[G_i^m]$, $f^{m_1} = Vector[f_i^{m_1}]$, $f^{m_2} = Vector[f_i^{m_2}]$, $w^m = Vector[w_i^m]$, and $\eta = Vector[\eta_i]$ can be obtained by Assumption 1.

An additional fuzzy control U_F is introduced to alleviate the reaching phase and to reduce the tracking errors more, compared with the traditional SMC and the saturation function (SMC-Sat). This fuzzy control U_F is developed according to the stability analysis explained in the following.

We call the controller (10) a "Fuzzy sliding mode controller with saturation *(FSMCS)*".

Substituting the presented controller (10)~(12) into (9) and adopting Property 1, $\dot{V}$ is obtained as

$$\dot{V} = s^T\Big[\ \Delta M(C\dot{e} - \ddot{q}_d) - M^m|C\dot{e} - \ddot{q}_d| sat(\frac{s}{\Phi}) + \Delta B(s - \dot{q}) - B^m|s - \dot{q}| sat\left(\frac{s}{\Phi}\right) - \Delta G - G^m sat\left(\frac{s}{\Phi}\right) + w - w^m sat(\frac{s}{\Phi}) - \Delta F - (f^{m_1} + f^{m_2}|\dot{q}|) sat(\frac{s}{\Phi}) - \eta sat(\frac{s}{\Phi})\ \Big] + s^T U_F. \tag{14}$$

The stability analysis is divided into two parts as follows and the stability is proved under Assumption 1.

1) *Case 1 (Outside of the boundary layer)*: $|s_i| > \Phi_i$, i=1,2,⋯n.

$$\dot{V} \le -\sum_{i=1}^{n} \eta_i |s_i| + s^T U_F = -\sum_{i=1}^{n} \eta_i |s_i| + \sum_{i=1}^{n} s_i U_{Fi}. \tag{15}$$

A fuzzy controller is designed in order that $s^T U_F = \sum_{i=1}^{n} s_i U_{Fi} < 0$. As a result, $\dot{V} < 0$ and thus outside the boundary layer, the asymptotic stability of s, e and $\dot{e}$ are guaranteed.

Outside the boundary layer, it is found that the reaching phase can be decreased by inserting a fuzzy control U_F, compared with the traditional SMC.

2) *Case 2 (Inside of the boundary layer):* $|s_i| \le \Phi_i$, i=1,2,⋯n.

The time derivative of V is finally bounded as

$$\dot{V} \le \sum_{i=1}^{n} \psi_i \left(|s_i| - \frac{s_i^2}{\Phi_i} \right) - \sum_{i=1}^{n} \frac{\eta_i}{\Phi_i} s_i^2 + \sum_{i=1}^{n} s_i U_{Fi} . \quad (16)$$

where $\psi_i = \sum_{j=1}^{n} M_{ij}^m \left| C\dot{e} - \ddot{q}_d \right|_j + \sum_{j=1}^{n} B_{ij}^m \left| \dot{q}_j \right| \left| s_j - \dot{q}_j \right|$ $+ G_i^m + f_i^{m_1} + f_i^{m_2} |\dot{q}_i| + w_i^m$.

Inside the boundary layer, the performance of the tracking errors can be improved by U_F.

Remark 1: When $s^T U_F$ comes to be negatively very large, (16) may be negative and then s may converge to zero asymptotically. At this time, the chattering may occur, however, the tracking error can be improved much more. Through this result, it is observed that the tracking errors in the proposed FSMCS can be made to be smaller than those in the traditional SMC with the saturation function (SMC-Sat) by adding a fuzzy control input U_F. □

The sliding surface s is fuzzified, because the condition of $s^T U_F < 0$ above is used in developing the fuzzy rules. The fuzzification and fuzzy inference of the presented fuzzy control are performed by the Mamdani method and the fuzzy control input U_F is obtained by the centroid method for defuzzification. The input and output membership functions used in the fuzzy inference are shown in Fig. 1 and Fig. 2, respectively. Table 1 shows the fuzzy If-Then rules.

To ensure a speedy fuzzy inference, the fuzzy sets include "N, P, B, M, S, ANZ, APZ" which represent "Negative, Positive, Big, Medium, Small, Almost Negative Zero, Almost Positive Zero". The value $\overline{U}$ used in the output membership function is designed, as the average value of u_s, in order to push the system state to the sliding surface.

Table 1: Fuzzy Rules

s	NB	NM	NS	ANS	APS	PS	PM	PB
U_F	PS	PM	PB	PB	NB	NB	NM	NS

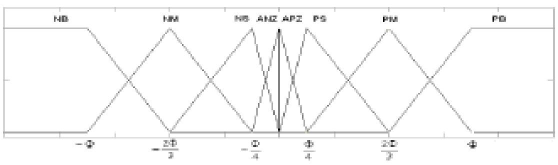

Fig. 1. Membership functions of the input s of FLC

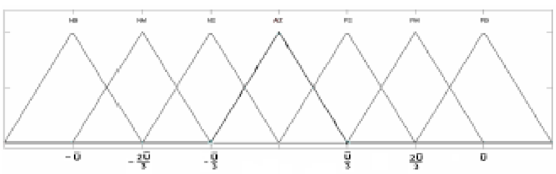

Fig. 2. Membership functions of the output U_F of FLC.

In [9], a linear uncertain system was studied to develop a FSMC. The presented work is performed for a nonlinear robot system with parameter variations, frictions and disturbances. The controller (10) shows the systematic and feasible design for the robust tracking of robot manipulators.

VI. SIMULATION

The proposed controller is applied to a two-link planar robotic manipulator with horizontal motion.

The desired trajectory is chosen as $q_{d_1} = q_{d_2} = \sin(0.8373t)(rad)$. During the control process, the manipulator lifts up an unknown load and therefore the dynamic parameters vary. The external disturbances which always inserted into each joint are $w_1 = w_2 = \sin(100t)(rad)$. The friction forces at each joint include the viscous friction, dynamic friction and static friction. The control gains and the width of boundary layer are selected as $c_1 = c_2 = 2$, $\Phi_1 = \Phi_2 = 0.8$, $\eta = 10$. The initial position and velocities are $\theta(0) = \phi(0) = 0(rad)$, $\dot{\theta}(0) = \dot{\phi}(0) = 0(rad/\sec)$.

Table 2: Real and Nominal Parameters

Parameters	Real Values	Nominal Values	Small Variations	Large Variations
m_1 (kg)	0.5	0.25		
m_2 (kg)	0.5	0.25	0.5	5.75
J_1 (kg.m^2)	0.25	0.125		
J_2 (kg.m^2)	0.25	0.125	0.175	0.625
l_1 (m)	1			
l_2 (m)	0.8			
g (m/s^2)	9.8			
f_{v1}	1.5	0.75		
f_{v2}	1.5	0.75		
k_1	0.2	0.1		
k_2	0.2	0.1		
$\overline{k}_{s1}$	0.1	0.05		
$\overline{k}_{s2}$	0.1	0.05		

m_1, J_1, r_1, f_{v1}, k_1 and $\overline{k}_{s1}$ are mass, moment of inertia, link length, coefficient of viscous friction, coefficient of dynamic friction, and coefficient of static friction of link 1, respectively. Similarly, the symbols for link 2 are also defined, respectively.

While the 2-link arm lifts up an unknown load, the robot parameter can be varied, such as $m_2 \to (m_2 + \Delta m_2)$ and $J_2 \to (J_2 + \Delta J_2)$. In this case, the value of parameters change at $t = 5(\sec)$ and $t = 10(\sec)$. The reason is that at first the manipulator lifts up a small load just after $5(\sec)$. Then the 2-link planar robot continuously lifts up a larger load at 10(sec) of the total execution time 15(sec).

Moreover, a few knowledge of the dynamic parameters is used in the controller. Hence, the nominal value is set as a half of real value.

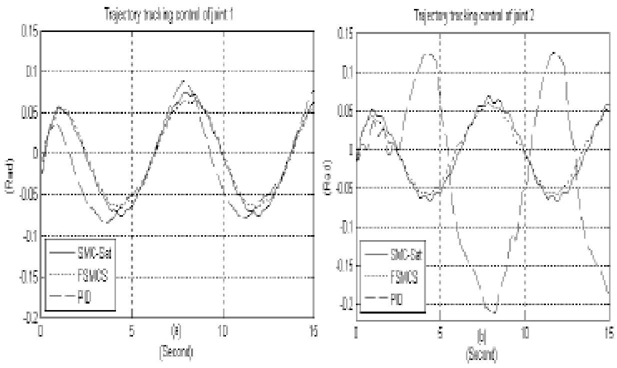

Fig. 3. Comparison of tracking position errors in joint 1 and joint 2 : SMC-Sat, FSMCS and PID controller

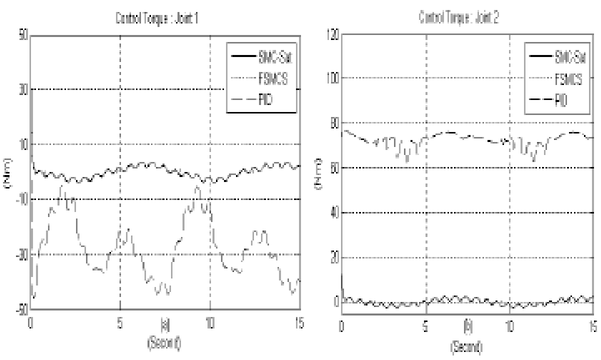

Fig. 4. Comparison of control torques in joint 1 and joint 2: SMC-Sat, FSMCS and PID controller

Table 3 summarizes a numerical comparison of the three control schemes, with maximum of absolute value and root-mean-square (rms) value of the tracking errors and control torques. The PID controller includes $K_P = 35, K_I = 50, K_D = 10$. From these comparative simulation results, it is found that the proposed control scheme is superior to the traditional SMC-Sat and PID.

Table 3: Comparison of the errors and control torques for each control scheme

Control Method	Link	Error (max) [rad]	Error (rms) [rad]	Control Torque (max) [Nm]	Control Torque (rms) [Nm]
SMC-Sat	1	0.0760	0.0488	42.8100	3.8858
	2	0.0679	0.0433	32.1900	2.0664
FSMCS	1	0.0657	0.0447	44.1452	3.9508
	2	0.0596	0.0375	33.5252	2.1087
PID Controller	1	0.0893	0.0531	45.8100	29.8455
	2	0.2125	0.1104	92.1900	72.6521

Consequently, we have found that the FSMCS scheme for robot manipulators is feasible and robust to the uncertainties such as parameter variations, unknown frictions and external disturbances through the simulations.

V. CONCLUSION

In this paper, a systematic and feasible design which combines the features of SMC and FLC has been presented. The proposed controller can alleviate the reaching phase and reduce the tracking errors by employing a fuzzy tuning scheme. It is found that the presented control scheme is valid and robust against the uncertainties such as parameter variations and disturbances. Through the comparative numerical simulations, the superiority of the proposed control scheme has been found.

REFERENCE

[1] B. W. Bekit, J. F. Whidborne and L. D. Seneviratne (1997), Fuzzy sliding mode control for a robot manipulator, Proc. of the IEEE International Sym. on Computational Intelligence in Robotics and Automation (CIRA'97), 1997, pp. 320-325.

[2] H. Foust, R. Bakeer, S. Drakunov (2005), Sliding mode control applied to an ultrafiltration process, Proc. of IEEE Int. Conf. on Control Applications (CCA 2005), August 2005, pp. 49-54.

[3] A. T. Araiza, J. L. M. Medina (1995), Variable structure with sliding mode controls for dc motors, Tech. Proc. of the IEEE Power Electronics Congress (CIEP'95), Oct. 1995, 26-28.

[4] J. S. Glower, J. Munighan (1997), Designing fuzzy controllers from a variable structure standpoint, IEEE Trans. on Fuzzy Systems, 5(1):138-144.

[5] O. Kaynak, K. Erbatur and M. Ertugrul (2001), The fusion of computationally intelligent methodologies and sliding mode control- A survey, IEEE Trans. on Industrial Electronics, 48(1):4-17.

[6] J. Y. Huang, W. Gao, and J. C. Hung (1993), Variable structure control: A survey, IEEE Trans. on Industrial Electronics, 40(1):2-22.

[7] F. Esfandiari and H. K. Khalil (1991), Stability analysis of a continuous implementation of variable structure control, IEEE Trans. on Automatic Control, 36(5):616-620.

[8] M. Chen, Y. Hwang and M. Tomizuka (2002), A state-dependent boundary layer design for sliding mode control, IEEE Trans. on Automatic Control, 47(10):1677-1681.

[9] Q. P. Ha et al (2001), Fuzzy sliding mode controllers with applications, IEEE Trans. on Industrial Electronics, 48(1):38-46.